VISION SCIENCE I

Monocular Sensory Processes of Vision:

Color Vision

Colorimetry
Colorimetry

A. Color mixing and color matching
B. Grassman’s laws
C. The unit trichromatic equation
D. Chromaticity diagrams
   1. The spectrum locus
   2. Trichromatic coordinates
E. The CIE chromaticity diagram
   1. Dominant wavelength
   2. Excitation purity
   3. Color mixtures
   4. Complements
   5. Non-spectral colors
Color mixture and color matching

- Color science is, primarily, the study of subjects’ discriminations when different colors are added by physical imposition or temporal alteration (additive mixtures).
Colorimetry

- All colors can be matched by varying the intensities of three primary stimuli (usually red, green and blue colors).
- If only three primaries are required to match all colors, then only three photopigments are necessary - color vision is trichromatic.
- Colorimetry is the science of color, involving the rules for matching, the mathematical specification of color, and the relationships between perceptual and physical variables.
Color Matching

Example: Red + Green + Blue $\equiv$ White

- Primaries do not need to be specific wavelengths or even monochromatic.

- Primaries must be unique - any one cannot be matched by a combination of the other two.

- Metamers are stimuli that are composed of different color components, but appear identical with respect to hue, saturation and brightness.

- Rayleigh match is a specific metameric match - the combination of red + green monochromatic primaries to match a monochromatic yellow, i.e., Red + Green $\equiv$ Yellow.
Grassman’s Laws of Color Mixture

• Grassman (1853) proposed that the additive mixture of colors could be treated mathematically as an addition of vectors.

• Grassman’s first law - Three independent variables are sufficient and necessary to specify a colored stimulus.
  1) Dominant wavelength
  2) Purity
  3) Photometric magnitude (brightness).
Grassman’s Laws of Color Mixture

Grassman (1853) proposed that the additive mixture of colors could be treated mathematically as an addition of vectors.

1. First law - Three independent variables are sufficient and necessary to specify a colored stimulus.
2. Second law - If one component of a two component mixture is changed and the other remains constant, then the color of the mixture changes.
   - Complementary colors
   - Non-Complementary colors
3. Third law - Lights of the same color produce identical effects in a mixture, regardless of their spectral composition.
   - Additive mixtures
   - Subtractive mixtures
4. Fourth law - The total intensity of a mixture is the sum of the intensities of the separate components in the mixture.
**Grassman’s Laws of Color Mixture**

Grassman’s second law - If one component of a two component mixture is changed and the other remains constant, then the color of the mixture changes.

- First corollary to the second law.

1. For every color there is a complementary color, which when mixed in the correct amount gives a gray or white, and if mixed in any other proportion gives an unsaturated hue of the stronger component.

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R  R > G  R = G  R < G  G
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Grassman’s Laws of Color Mixture

Grassman’s second law - If one component of a two component mixture is changed and the other remains constant, then the color of the mixture changes.

• Second corollary to the second law.

2. The mixture of any two colors that are not complementary gives an intermediate color varying in hue with the relative amounts of the original colors and varying in saturation with the nearness or remoteness of the colors.

R → R > B → R = B → R < B → B
Grassman’s Laws of Color Mixture

Grassman’s third law - Lights of the same color produce identical effects in a mixture, regardless of their spectral composition.

• First corollary to the third law.

1. Color mixtures are additive - two lights of the same color added to two other lights of the same color produce mixtures that are also the same color.

   Example: If color A ≡ color B and color C ≡ color D, then color A + color C ≡ color B + color D
Grassman’s Laws of Color Mixture

Grassman’s third law - Lights of the same color produce identical effects in a mixture, regardless of their spectral composition.

• Second corollary to the third law.

2. Color mixtures are subtractive - two lights that are of the same color can each be subtracted from mixtures of the same colors and leave mixtures that match.

Example: If color A \equiv color B and color C \equiv color D, then color A - color C \equiv color B - color D
Grassman’s Laws of Color Mixture

Grassman’s fourth law - The total intensity of a mixture is the sum of the intensities of the separate components in the mixture.

Example: If color A + color B ≡ color C, then
(color A + color B) / 2 ≡ color C / 2
or (color A + color B) * 2 ≡ color C * 2

- Grassman’s fourth law is also known as Abney’s law.
- Abney’s Law – an empirical law stating that if 2 colored stimuli, A and B, are perceived to be of equal brightness and 2 other color stimuli, C and D, are perceived to be of equal brightness, then the additive mixtures of A with C and B with D will also be perceived to be of equal brightness.
Grassman’s Laws of Color Mixture

• Grassman’s laws provide a method of defining color mixtures by simple algebra.

If \( L_1(C_1) = U_1 R + V_1 G + W_1 B \)
and \( L_2(C_2) = U_2 R + V_2 G + W_2 B \)

then \( L_1(C_1) + L_2(C_2) = (U_1 + U_2) R + (V_1 + V_2) G + (W_1 + W_2) B \)

• Shortcomings:
  1) Color mixture representation is in 3D space.
  2) The intensity units for the primaries and the mixtures are arbitrary and do not relate to the perceptual brightness of a stimulus.
The “unit trichromatic equation”

• The unit trichromatic equation is derived from color mixture equations to provide relative (rather than absolute) intensity coefficients for each primary and to reduce trichromatic color mixture data to a two-dimension co-ordinate system.

• Each intensity value is divided by the sum of the intensities of the primaries. For example, the mixture data for a standard white stimulus (S)

\[ L_s(S) \equiv U_s R + V_s G + W_s B \]

let \( L = L_s / (U_s + V_s + W_s) \); \( x = U_s / (U_s + V_s + W_s) \)

\( y = V_s / (U_s + V_s + W_s) \); \( z = W_s / (U_s + V_s + W_s) \)

• For any colored stimulus, \( C = xR + yG + zB \), where \( x + y + z = 1 \).

• For a white stimulus, \( S = .33R + .33G + .33B \)
Chromaticity Diagrams

- A chromaticity diagram is a representation of 3-D color mixture space on a 2-D plot by plotting colors with coefficients for the R and G primaries.
- Color space is defined by the color triangle.
- For $C = (x,y)$, $z = 1 - (x + y)$.
- For the white point (W), $x = .33$, $y = .33$ and, therefore, $z = .33$.
The Spectrum Locus

• Spectrum locus - The locations of the spectral colors on a chromaticity diagram.

• In this example of Wright’s measurements with monochromatic primaries of 650, 530, and 460 nm.

• All visible stimuli must lie inside the spectrum locus.
Trichromatic Coordinates for the Spectrum Locus

- The average color matching coefficients for subject’s with normal color vision
- For most of the wavelengths, one of the coefficients is negative because the spectrum locus lies outside of the color triangle.
- Often it was incorrectly assumed that the coefficient functions should represent the action spectral of the cone photopigments.
**Negative Trichromatic Coefficients**

- An application of the second corollary to Grassman’s second law.

- To match the Color M, a certain amount of the primary R must be added to the sample M, to desaturate the sample. Then, the desaturated sample P can be matched by the appropriate combination of primaries.

  \[ C - x_1 R = x_2 R + yG + zB \]

  \[ C = (x_1 - x_2)R + yG + zB \]
The CIE Chromaticity Diagram

• A revised chromaticity diagram was developed to eliminate three problems with Wright’s system.

  1) the negative chromaticity coefficients.
  2) attempts to assign physiological significance to the trichromatic coefficient functions.
  3) the color specification lacked information on the relative brightness of a color.

• The CIE (International Commission on Illumination) system was based on a transformation of color mixture data to a set of imaginary primaries, which eliminated negative coefficients and the assumption of physiological significance.

• One of the imaginary primaries was the human photopic luminous efficiency function ($V_\lambda$) so that one coefficient would reflect relative brightness information.
The Primaries for the CIE Chromaticity Diagram

- The CIE system is also called the XYZ system because the primaries are designated as X, Y, and Z.
- The X primary, the long wavelength primary, is bimodal, unlike any real photopigment.
- The Y primary, the middle wavelength primary, is the photopic luminosity function.
The CIE Chromaticity Diagram

- The CIE chromaticity diagram is based on the unit trichromatic equation for the X, Y, and Z primaries.
- The spectrum locus is straight or concave with respect to color space.
- Any mixture of real colors must lie on or inside of the spectrum locus. Colors lying outside of the spectrum locus are imaginary colors.
Perceptual Color Relationships in the CIE Chromaticity Diagram

- Any colored stimulus can be specified in terms of its CIE chromaticity coordinates.
Perceptual Attributes - Dominant Wavelength

- In addition to brightness (Y coefficient), a stimulus (M) can be described by its dominant wavelength and excitation purity.

- The dominant wavelength of M is determined by the wavelength on the spectrum locus that is intersected by a line drawn from white through the sample (about 540 nm in the example).
Perceptual Attributes - Excitation Purity

- The excitation purity of sample M is specified by the ratio of the distance of white to the sample with respect to the distance of white to the dominant wavelength on the spectrum locus (about 0.6 or 60% in the example).

- Excitation purity is a physical specification and does not correlate with perceptual saturation.

- Colorimetric purity is a psychophysical measure of perceived saturation.
Perceptual Attributes - Brightness

- The y-primary, the middle wavelength primary, is the photopic luminosity function.
- The y-primary tristimulus coefficient represents the relative photopic luminosity of any stimulus that can be located on the CIE chromaticity diagram.
Perceptual Attributes - Brightness

- The brightness of sample M is specified by the y-primary coefficient (about 0.57).
- The y-primary is the $V_\lambda$ for an equal-energy spectrum, which has a peak at about 520 nm.
- In the CIE system the x- and z-primaries have zero luminous intensity. Colors on the x-axis or z-axis (on the alychne) are imaginary colors with zero luminous intensity.
Perceptual Attributes - Color Mixtures

- An application of Grassman’s second law of color mixture.
- The mixture of any two colors will produce a new color that lies on a straight line connecting the two colors.
- The location along the line will be determined by a “center of gravity” principle, i.e., an inverse proportion of the intensities. (example, if C1 = 20 luminance units (lu) and C2 = 10 lu, the ratio of distances C1-C3 = 1/3 C1-C2, with 30 lu intensity).
Perceptual Attributes - Complementary Colors

- Colors are complementary if they lie on a line that passes through white, but are on opposites of the white point.
- In the example, colors C1 and C2 are complements. Spectral colors at 480 and 580 nm are also complements.
Perceptual Attributes - Non-Spectral Colors

- Colors lying in a triangle formed by white and the ends of the spectrum locus are non-spectral, that is, they do not have a dominant wavelength.

- The non-spectral colors are the purples (redish-blues) and are designated by their complements.

- In the example, the dominant wavelength of color C1 is designated as 510C (complement of 510 nm).
Perceptual Attributes - Discriminability of Colors

- A feature of color mixture space that would be highly desirable - just-noticeable-differences (JNDs) in chromaticity should be equal throughout color mixture space, i.e., circles of equal diameter at all locations of the chromaticity diagram.

- MacAdam’s investigations showed that JND’s in chromaticity are elliptical (MacAdam’s ellipses) and differ in size throughout color mixture space.
Colorimetry

Recap

• Colorimetry – perceptual attributes of additive color mixtures.
• Primaries
• Color matching (metameres)
• Grassman’s Laws
• The unit trichromatic equation
• Chromaticity diagrams
• Perceptual attributes represented in CIE space